Theoretical Computer Science 824–825 (2020) 81–91

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Restricted connectivity and good-neighbor diagnosability of split-star networks

Limei Lin [a,c,*], Yanze Huang [a,b], Xiaoding Wang [a,c], Li Xu [a,c]

[a] College of Mathematics and Informatics, Fujian Normal University, Fuzhou, Fujian, 350117, PR China
[b] School of Mathematics and Physics, Fujian University of Technology, Fuzhou, Fujian, 350118, PR China
[c] Key Laboratory of Network Security and Cryptology, Fujian Normal University, Fuzhou, Fujian, 350007, PR China

ARTICLE INFO

Article history:
Received 30 October 2019
Received in revised form 20 February 2020
Accepted 18 April 2020
Available online 27 April 2020
Communicated by S.-Y. Hsieh

Keywords:
Good-neighbor diagnosability
Computer network reliability
Fault tolerance
Restricted connectivity
Split-star networks

ABSTRACT

The restricted connectivity and the g-good-neighbor diagnosability are two important indicators of the robustness for a multi-processor system in presence of failing processors. The g-good-neighbor diagnosability of a graph guarantees that the number of fault-free neighbors of every fault-free vertex is greater or equal to g in the graph. We first establish the 3-restricted connectivity of an n-dimensional split-star network S_n^2. Then we propose the upper bound of the $\{1, 2, 3\}$-good-neighbor diagnosability of S_n^2 under the MM* model. Moreover, we show that when deleting two indistinguishable good-neighbor faulty vertex-sets from S_n^2, the remaining connected subgraph has no isolated vertex. Furthermore, we give a complete proof for the lower bound of the $\{1, 2, 3\}$-good-neighbor diagnosability of S_n^2, and prove that the lower and upper bounds of the $\{1, 2, 3\}$-good-neighbor diagnosability of S_n^2 are accurate.

1. Introduction

Failure of processor is inevitable in a multiprocessor system with a great quantity of processors. A novel diagnostic model, the *comparison model* (*MM model*), was established by Malek and Maeng [26]. Under the MM model, a test contains a vertex u and two of its neighbors v, w. The vertex u sends the same input to v, w and then compares their feedbacks. If u is faulty, then the test result is not reliable. When u is fault-free, the test result is 0 if v, w are both fault-free and 1 otherwise. In 1992, Sengupta and Dahbura [28] proposed the MM* model, which is a special MM model. Under the MM* model, every vertex must compare every pair of its neighbors. Fan [11,12] proposed the diagnosability of the Möbius cubes and crossed cubes under the MM* model. Moreover, Chen and Hsieh [3] gave the (t, k)-diagnosis for component-composition graphs under the MM* model. To better reflect a network's true self fault-diagnosing capability, Chang and Hsieh [2] studied the conditional diagnosability of augmented cubes.

In 2012, Peng et al. [27] proposed the *g-good-neighbor diagnosability* of a graph, which assumes that the number of fault-free neighbors of every fault-free vertex is greater or equal to g in the graph under the PMC model. Since this conception was raised, a lot of works have been done on it. Yuan et al. [34] gave the g-good-neighbor diagnosability of k-ary n-cubes. In 2016, Wang et al. [29] proposed the g-good-neighbor diagnosability of n-hypercubes under the MM* model. We [25] established the g-good-neighbor diagnosability of arrangement graphs by exploring the known g-restricted connectivity and the size of K_{g+1}. Xu et al. [33] also proposed the g-good-neighbor diagnosability of (n, k)-star graphs by combining

* Corresponding author at: College of Mathematics and Informatics, Fujian Normal University, Fuzhou, Fujian, 350117, PR China.
E-mail address: putianlinlimei@163.com (L. Lin).

https://doi.org/10.1016/j.tcs.2020.04.015
0304-3975/

the known g-restricted connectivity with the size of the $(g+1)$-dimensional complete graph (K_{g+1}) for $0 \le g \le n-k$. In 2017, Wei and Xu [31] extended the result for (n,k)-star networks to $0 \le g \le n-1$. Gu et al. [13] gave a short note on the $\{1,2\}$-good-neighbor diagnosability of balanced hypercubes. Guo et al. [15] studied the g-good-neighbor diagnosability of crossed cubes. Li and Lu [21] proposed g-good-neighbor conditional diagnosability of star graphs. In 2019, we studied the g-good-neighbor diagnosability for exchanged hypercubes [35] and alternating group graphs [18].

Moreover, there exist some general works about the g-good-neighbor diagnosability. Gu et al. [14] studied the 1-good-neighbor diagnosability of some regular graphs. Then, Wei and Xu [32] explored the $\{1,2\}$-good-neighbor conditional diagnosability of some special regular graphs, including BC graphs, folded hypercubes and four classes of Cayley graphs. Hu et al. [17] established the equal relationship between g-good-neighbor diagnosability under the PMC model and MM* model. Furthermore, in 2018, we [22] established the relationship between g-restricted connectivity and g-good-neighbor fault diagnosability of general triangle-free regular networks. Then, Cheng [4], Wang et al. [30] and Cheng et al. [10] also provided a relationship between g-good-neighbor diagnosability and g-restricted connectivity in regular graphs with different conditions.

These related works do not involve the study of the g-good-neighbor diagnosability for the split-star networks. In this paper, we aim to solve the problem for the structure called an n-dimensional *split-star network* S_n^2, which was proposed by Cheng et al. [7] as an alternative to the popular n-dimensional star graph S_n [1]. The construction of the split-star network is rather different from the hypercubes, k-ary n-cubes, star graphs, arrangement graph, (n,k)-star graphs, balanced hypercubes, crossed cubes, exchanged hypercube and alternating group graphs, and it has many advantages compared with these above networks. Moreover, the split-star network has many 3-cycles in it, which is different from our reference [22]. The 3-good-neighbor diagnosability and 3-restricted connectivity also have not been presented. Hence, the above general related works for g-good-neighbor diagnosability are not applicable to the work of this paper. We will give a complete method for 3-restricted connectivity. Moreover, we will propose the $\{1,2,3\}$-good-neighbor diagnosability by considering a unified approach.

To highlight our contributions, we summarize them as follows:

- We establish the 3-restricted connectivity of an n-dimensional split-star network S_n^2.
- We propose the upper bound of the $\{1,2,3\}$-good-neighbor diagnosability of S_n^2 by construction under the MM* model.
- When deleting two indistinguishable good-neighbor faulty vertex-sets from S_n^2, the remaining connected subgraph has no isolated vertex.
- We prove that the upper and lower bounds of the $\{1,2,3\}$-good-neighbor diagnosability of S_n^2 are accurate.

This paper is divided into four parts. Section 2–Section 5 establish the preliminaries used throughout the paper, the 3-restricted connectivity of S_n^2, the $\{1,2,3\}$-good-neighbor diagnosability of S_n^2 under the MM* model, and the conclusion, respectively.

2. Preliminaries

We give some preparing works to establish the 3-restricted connectivity and the g-good-neighbor diagnosability of S_n^2 under the MM* model.

2.1. Terminology

In this subsection, we give some basic terminologies, which were included in our previous works [18],[22],[23],[24],[25] and [35].

- $G=(V(G),E(G))$: a graph with the vertex-set $V(G)$ and the edge-set $E(G)$.
- $u \in V(G)$: a vertex u from the vertex-set $V(G)$.
- $uv \in E(G)$: an edge uv from the edge-set $E(G)$.
- $|A|$: the size of a set A.
- $M \subseteq G$: M is a subgraph of G, in which $V(M) \subseteq V(G)$ and $E(M) \subseteq E(G)$.
- $G[B]$: an induced subgraph of G by the vertex-set B with $V(G[B])=B$ and $E(G[B])=\{xy \mid xy \in E(G), x,y \in B\}$.
- $\bigcup_{i=1}^m G_i = G[\bigcup_{i=1}^m V(G_i)]$ and $\bigcap_{i=1}^m G_i = G[\bigcap_{i=1}^m V(G_i)]$.
- $G-B$: a subgraph of G by deleting all vertices of the vertex-set B from a graph G and all edges connecting at least one vertex in the vertex-set B.
- $A-B$: a set of vertices who are in the vertex-set A and not in the vertex-set B.
- $E[V(G_1),V(G_2)]=\{xy \mid x \in V(G_1) \text{ and } y \in V(G_2)\}$.
- $N_G(x)=\{y \in V(G) \mid xy \in E(G)\}$.
- $N_G(B)=(\bigcup_{u \in B} N_G(x))-B$.
- $\delta(G)$: the minimum value of degrees of all vertices in a graph G.
- P_k (or C_k): a path (or cycle) with length k, called a k-path (or k-cycle).

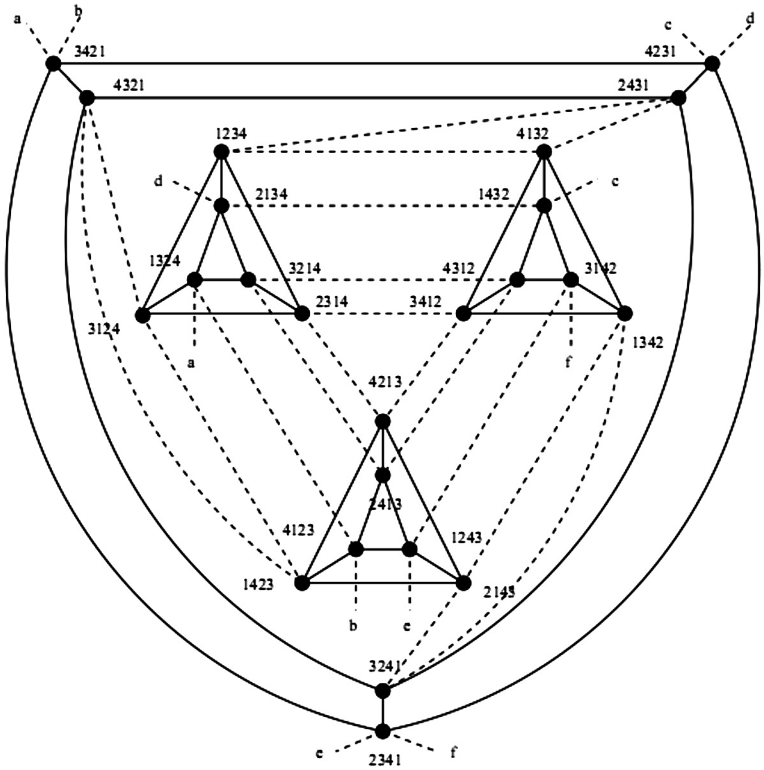

Fig. 1. The 4-dimensional split-star network S_4^2.

2.2. Split-star network

In subsection 2.2, we give the definition of the n-dimensional split-star network S_n^2 (Definition 1) and some basic properties of S_n^2 (Remark 1, Remark 2 and Lemma 1).

Definition 1. [7,8] Given two positive integers n and k with $n > k$, let $\langle n\rangle = \{1, 2, \dots, n\}$, and let $\mathbb{P}_n$ be a set of $n!$ permutations on $\langle n\rangle$. The n-dimensional split-star network, denoted by S_n^2, such that

- $V(S_n^2) = \mathbb{P}_n$;
- $E(S_n^2) = \{pq \mid p$ (resp. q) can be obtained from q (resp. p) by either a 2-exchange or a 3-rotation$\}$.

(1) A 2-exchange interchanges the symbols in 1st position and 2nd position.
(2) A 3-rotation rotates the symbols in 1st, 2nd and kth for some $k \in \{3, 4, \dots, n\}$.

Remark 1. For any vertex u and a fixed $k \in \{3, 4, \dots, n\}$, there are two 3-rotations $\binom{123\cdots k\cdots n}{2k3\cdots 1\cdots n}$ and $\binom{123\cdots k\cdots n}{k13\cdots 2\cdots n}$, so the vertex u has two neighbors by the 3-rotations for this k. Hence, S_n^2 is a $(1 + 2(n-2))$-regular graph with $n!$ vertices. We use $x = x_1x_2\cdots x_i\cdots x_n$ to denote a permutation where x_i is in ith position. Fig. 1 gives the structure of S_4^2.

Remark 2. For any $i \in \{1, 2, \dots, n-2, n-1, n\}$, let $S_n^{2:i} = S_n^2[V(S_n^{2:i})]$ where the set $V(S_n^{2:i}) = \{x_1x_2\cdots x_{n-2}x_{n-1}i \mid x_{j_1} \neq x_{j_2} \in \{1, 2, \dots, n-2, n-1, n\} - \{i\}$ where $1 \le j_1 \neq j_2 \le n-1\}$. Every vertex v in $S_n^{2:i}$ has exactly two neighbors not in $S_n^{2:i}$, who are called as the *external-neighbors* of v. A pair of elements x_i and x_j is called an inversion of x if $x_i < x_j$ whenever $i > j$.

The n-dimensional split-star network can be decomposed to two different networks by *even permutation* and the *odd permutation*, in which the former and the latter contain an even and an odd number of inversions, respectively. Let $S_{n,E}^2$ be an induced subgraph $S_n^2[V(S_{n,E}^2)]$ with $V(S_{n,E}^2) = \{u \mid u$ is an even permutation in $V(S_n^2)\}$. It can be found that $S_{n,E}^2$ is an n-dimensional alternating group graph AG_n [19]. Let $S_{n,O}^2$ be an induced subgraph $S_n^2[V(S_{n,O}^2)]$ with $V(S_{n,O}^2) = \{u \mid u$ is an odd permutation in $V(S_n^2)\}$. Hence, $S_{n,O}^2$ is isomorphic to $S_{n,E}^2$ by a 2-exchange and there are $n!/2$ disjointed edges between the subgraphs $S_{n,E}^2$ and $S_{n,O}^2$. Fig. 2 gives the subgraphs $S_{4,E}^2$ and $S_{4,O}^2$.

Lemma 1. [5,6,23] (1) *The n-dimensional split-star network S_n^2 is $(2n-3)$-regular and the connectivity of S_n^2 is $2n-3$ for $n \ge 2$.*
(2) *Every vertex in $S_n^{2:i}$ has two external-neighbors, which are in distinct subgraphs and adjacent.*

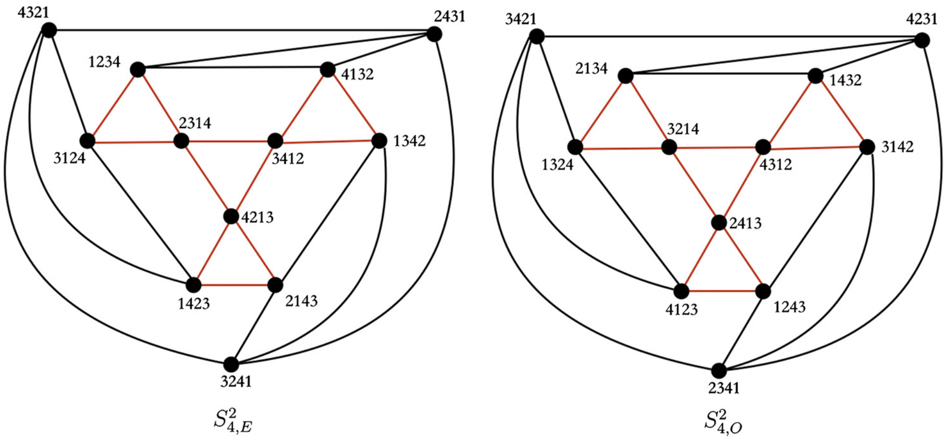

Fig. 2. The subgraphs $S^2_{4,E}$ and $S^2_{4,O}$.

(3) *Any two vertices in $S_n^{2:i}$ have different external-neighbors.*
(4) *Let $\zeta, \eta \in V(S_n^2)$. If $\zeta\eta \notin E(S_n^2)$, then $|N_{S_n^2}(\zeta) \cap N_{S_n^2}(\eta)| \leq 2$. If $\zeta\eta \in E(S_n^2)$, then $|N_{S_n^2}(\zeta) \cap N_{S_n^2}(\eta)| = 1$.*
(5) *There is one to one correspondence between the subgraph $S^2_{n,O}$ and the subgraph $S^2_{n,E}$.*

3. The 3-restricted connectivity of S_n^2

In this section, we will establish the sufficient conditions to determine the 3-restricted connectivity of S_n^2.

Definition 2. [20] Given a graph G, a subset D in $V(G)$ and an nonnegative integer g.
(1) If $G - D$ is disconnected, then D is called a vertex-cut of G. The maximal connected subgraph in $G - D$ is called a component.
(2) If D is a vertex-cut and $\delta(G - D) \geq g$, then D is called a g-restricted vertex-cut of G.
(3) The *g-restricted connectivity* of G, denoted by $\kappa^g(G)$, is the minimum cardinality over all g-restricted vertex-cuts of G. A g-restricted vertex-cut is called to be the minimum if the cardinality of the g-restricted vertex-cut is $\kappa^g(G)$.

We [24] have obtained the $\{1, 2\}$-restricted connectivity of S_n^2.

Lemma 2. [24] *Let S_n^2 be an n-dimensional split-star network, then the following properties hold.*
(1) *$\kappa^1(S_n^2) = 4n - 9$ ($n \geq 4$). Furthermore, let S be a 1-path in $S^2_{n,E}$, then $N_{S_n^2}(V(S))$ is a minimum 1-restricted vertex-cut of S_n^2.*
(2) *$\kappa^2(S_n^2) = 6n - 15$ ($n \geq 5$). Furthermore, let C_3 be a 3-cycle in $S^2_{n,E}$, then $N_{S_n^2}(V(C_3))$ is a minimum 2-restricted vertex-cut of S_n^2.*

Because the n-dimensional split-star network can be viewed as "companion graphs" of the n-dimensional alternating group graphs AG_n. We introduce the following some basic properties, which can be used to study the 3-restricted connectivity of S_n^2.

Lemma 3. [9] *Let D be a vertex-cut of AG_n ($n \geq 5$) such that $|D| \leq 6n - 20$. Then, $AG_n - D$ satisfies one of the following conditions.*
(1) *$AG_n - D$ has two components, one of which is a singleton, or an edge;*
(2) *$AG_n - D$ has three components, two of which are singletons.*

Lemma 4. [16] *Let D be a subset of $V(AG_n)$ ($n \geq 5$) such that $|D| \leq 6n - 19$. Then, $AG_n - D$ satisfies one of the following conditions.*
(1) *$AG_n - D$ is connected;*
(2) *$AG_n - D$ has two components, one of which is a singleton, an edge or a 2-path;*
(3) *$AG_n - D$ has three components, two of which are both singletons, respectively.*

Lemma 5. [18] *Let AG_n ($n \geq 5$) be an n-dimensional alternating group graph. Then, the 2-restricted connectivity is $\kappa^2(AG_n) = 6n - 18$. Furthermore, let $C_3 = \{u, v, w\}$ be a 3-cycle in AG_n ($n \geq 5$) such that $u = 1234 \cdots i \cdots n$, $v = 2431 \cdots i \cdots n$ and $w = 4132 \cdots i \cdots n$. It can be deduced that $N_{AG_n}(V(C_3))$ is a minimum 2-restricted vertex-cut of AG_n.*

Theorem 1. *Let S_n^2 be an n-dimensional split-star network. Let S_3^2 be a 3-dimensional split-star subnetwork in S_n^2 for $n \geq 5$. Then the 3-restricted connectivity $\kappa^3(S_n^2) = 12n - 36$ and $N_{S_n^2}(V(S_3^2))$ is a minimum 3-restricted vertex-cut of S_n^2 for $n \geq 5$.*

Proof. First, we prove that $\kappa^3(S_n^2) \le 12n - 36$.

Let S_3^2 be a 3-dimensional split-star subnetwork in S_n^2 ($n \ge 5$). Without loss of generality, assume that $V(S_{3,E}^2) = \{u, v, w\}$ such that $u = 1234\cdots i\cdots n$, $v = 2431\cdots i\cdots n$ and $w = 4132\cdots i\cdots n$ and $V(S_{3,O}^2) = \{u', v', w'\}$ such that $u' = 2134\cdots i\cdots n$, $v' = 4231\cdots i\cdots n$ and $w' = 1432\cdots i\cdots n$. Obviously, $S_{3,E}$ is a minimum 2-restricted vertex-cut of $S_{n,E}^2$ by Lemma 5 and the fact that $S_{n,E}^2$ is an n-dimensional alternating group graph AG_n. Hence, $|N_{S_{n,E}^2}(V(S_{3,E}^2))| = 6n - 18$ by Lemma 5. Moreover, every vertex of $S_{n,E}^2 - N_{S_{n,E}^2}(V(S_{3,E}^2)) - V(S_{3,E}^2)$ has at least two neighbors in $S_{n,E}^2 - N_{S_{n,E}^2}(V(S_{3,E}^2)) - V(S_{3,E}^2)$ and every vertex of $V(S_{3,E}^2)$ has exactly two neighbors in $V(S_{3,E}^2)$. Because $S_{n,E}^2$ is isomorphic to $S_{n,O}^2$, $S_{3,O}^2$ is a minimum 2-restricted vertex-cut of $S_{n,O}^2$ by Lemma 5. Hence, $|N_{S_{n,O}^2}(V(S_{3,O}^2))| = 6n - 18$ by Lemma 5. Moreover, every vertex of $S_{n,O}^2 - N_{S_{n,O}^2}(V(S_{3,O}^2)) - V(S_{3,O}^2)$ has at least two neighbors in $S_{n,O}^2 - N_{S_{n,O}^2}(V(S_{3,O}^2)) - V(S_{3,O}^2)$ and every vertex of $V(S_{3,O}^2)$ has exactly two neighbors in $V(S_{3,O}^2)$.

By Lemma 1 (5), there is one to one correspondence between the subgraph $S_{3,O}^2$ (resp. $S_{n,O}^2 - N_{S_{n,O}^2}(V(S_{3,O}^2)) - V(S_{3,O}^2)$) and the subgraph $S_{3,E}^2$ (resp. $S_{n,E}^2 - N_{S_{n,E}^2}(V(S_{3,E}^2)) - V(S_{3,E}^2)$). Thus, every vertex of $S_n^2 - N_{S_n^2}(V(S_3^2)) - V(S_3^2)$ has at least three neighbors in $S_n^2 - N_{S_n^2}(V(S_3^2)) - V(S_3^2)$ and every vertex of $V(S_3^2)$ has exactly three neighbors in $V(S_3^2)$. Therefore, $N_{S_n^2}(V(S_3^2))$ is a 3-restricted vertex-cut of S_n^2. By Definition 2 (3),

$$\begin{aligned}\kappa^3(S_n^2) &\le |N_{S_n^2}(V(S_3^2))| \\ &= |N_{S_{n,O}^2}(V(S_{3,O}^2))| + |N_{S_{n,E}^2}(V(S_{3,E}^2))| \\ &= 2 \times (6n - 18) \\ &= 12n - 36.\end{aligned} \tag{3.1}$$

Next, we prove that $\kappa^3(S_n^2) \ge 12n - 36$.

Suppose that $\kappa^3(S_n^2) \le 12n - 37$. Let D be a minimum 3-restricted vertex-cut of S_n^2, $|D| = \kappa^3(S_n^2) \le 12n - 37$. Let $D_O = D \cap V(S_{n,O}^2)$ and $D_E = D \cap V(S_{n,E}^2)$. Hence, $|D_O| + |D_E| = |D| \le 12n - 37$. Hence, there exists at least one of $|D_O|$ and $|D_E|$ with that the size is less than $6n - 18$. Without loss of generality, assume that $|D_E| \le 6n - 19$. By Lemma 4, $S_{n,E}^2 - D_E$ is connected; or has two components, one of which is a singleton, an edge or a 2-path; has three components, two of which are both singletons, respectively.

When $|D_O| \le 6n - 19$, by Lemma 4, $S_{n,O}^2 - D_O$ is connected; or has two components, one of which is a singleton, an edge or a 2-path; has three components, two of which are both singletons, respectively. When $S_{n,E}^2 - D_E$ is connected and $S_{n,O}^2 - D_O$ is connected, by Lemma 1 (5), there exactly $n!/2$ disjoint edges between $S_{n,O}^2$ and $S_{n,E}^2$. Because $n!/2 > 12n - 37$ for $n \ge 5$. Hence, there exists at least one edge between $S_{n,E}^2 - D_E$ and $S_{n,O}^2 - D_O$. It can be implied that $S_n^2 - D$ is connected, which contradicts that D is a 3-restricted vertex-cut of S_n^2.

Without loss of generality, assume that $S_{n,E}^2 - D_E$ is disconnected, there exists one vertex u such that its degree in $S_{n,E}^2 - D_E$ is less than two. By Lemma 1 (5), u has exactly one neighbor in $S_{n,O}^2$. Hence, u has at most two neighbors in $S_n^2 - D$. It contradicts that D is a 3-restricted vertex-cut of S_n^2.

When $|D_O| \ge 6n - 18$, we will deduce a contradiction. If $S_{n,E}^2 - D_E$ is disconnected, there exists one vertex u such that its degree in $S_{n,E}^2 - D_E$ is less than two. By Lemma 1 (5), u has exactly one neighbor in $S_{n,O}^2$. Hence, u has at most two neighbors in $S_n^2 - D$. It contradicts that D is a 3-restricted vertex-cut of S_n^2. If $S_{n,E}^2 - D_E$ is connected, let D'_E be a subset in $S_{n,O}^2$ such that there is one to one correspondence between D'_E and D_E. Let $D'_E \cap D_O = D'_O$. Hence, $|D'_O| \le |D'_E| = |D_E| \le 6n - 19$. If $|D'_O| = |D'_E|$, by Lemma 1 (5), $S_n^2 - D$ is connected. It contradicts that D is a 3-restricted vertex-cut of S_n^2. If $|D'_O| < |D'_E| \le 6n - 19$. By Lemma 3, $S_{n,O}^2 - D'_O$ is connected; or has two components, one of which is a singleton, or an edge; or has three components, two of which are singletons. By Lemma 1 (5), every vertex of $S_{n,O}^2 - (D'_E \cup D_O)$ connects to $S_{n,E}^2 - D_E$.

If there exists one vertex v in $D'_E - D'_O$ such that v can not be connected to $S_{n,O}^2 - D'_O$, then this vertex v must be the union of small components in $S_{n,O}^2 - D'_O$. Hence, v has at most one neighbor in $S_n^2 - D$. It contradicts that D is a 3-restricted vertex-cut of S_n^2. Hence, every vertex in $D'_E - D'_O$ can be connected to $S_{n,O}^2 - D'_O$. If every vertex in $D'_E - D'_O$ can be connected to $S_{n,O}^2 - D'_O - D_O$, then $S_n^2 - D$ is connected. It contradicts that D is a 3-restricted vertex-cut of S_n^2. Thus, there exists some vertices, say X, in $D'_E - D'_O$ such that X connects to $D_O - D'_O$. Therefore, $N_{S_{n,O}^2}(X) \subseteq D_O$ and the minimum degree of X is three. According to the structure of AG_n, $|X| \ge 9$ and $|D_E| \ge 9$. By the proof of Lemma 5, $|D_O| \ge |N_{S_{n,O}^2}(X)| \ge 8n - 28 + 6n - 18 - 5 = 14n - 51$. Hence, $|D| = |D_O| + |D_E| \ge 14n - 51 + 9 = 14n - 42 > 12n - 37 \ge |D|$, which is a contradiction.

Therefore, the 3-restricted connectivity $\kappa^3(S_n^2) = 12n - 36$ for $n \ge 5$. By Equation (3.1), $|N_{S_n^2}(V(S_3^2))| = 12n - 36 = \kappa^3(S_n^2)$. By Definition 2 (3), $N_{S_n^2}(V(S_3^2))$ is a minimum 3-restricted vertex-cut of S_n^2. $\square$

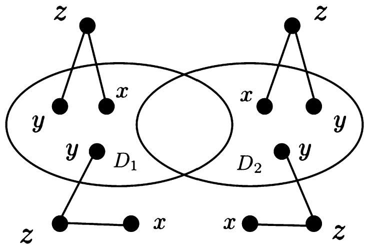

Fig. 3. Two subsets D_1 and D_2 in the graph G are distinguishable under the MM* model.

4. The {1, 2, 3}-good-neighbor diagnosability of S_n^2 under the MM* model

We first introduce the concept of the faulty vertex-set of a graph (Remark 3), the development of the good-neighbor diagnosability of a graph (Remark 4), the definition of the g-good-neighbor diagnosability of a graph (see Definition 3), the distinguishable faulty vertex-sets (Remark 5 and Lemma 6), which were included in our previous works [22],[25],[35].

Remark 3. The faulty vertex-set of a graph G is the set of all faulty vertices of G. A vertex could test another vertex if and only if there is an edge between them.

Remark 4. In 2012, Peng et al. [27] proposed the g-good-neighbor diagnosability of a graph by assuming that the number of fault-free neighbors of every fault-free vertex is greater or equal to g in the graph.

Definition 3. [27] Let G be a graph.

(1) A faulty vertex-set D is called a g-good-neighbor faulty vertex-set of G if $|N_G(x) \cap (V(G-D))| \geq g$ for $x \in V(G-D)$.

(2) A system G is g-good-neighbor t-diagnosable if D_1 and D_2 are distinguishable for any two g-good-neighbor faulty vertex-sets D_1 and D_2 with $|D_1| \leq t$, $|D_2| \leq t$.

(3) The g-good-neighbor diagnosability of a graph G under the MM* model, denoted by $t_g^m(G)$, is the maximum value of t such that G is g-good-neighbor t-diagnosable.

Remark 5. Sengupta and Dahbura [28] proposed an equivalent condition for that two subsets D_1 and D_2 in a graph G are distinguishable under the MM* model.

Lemma 6. [28] *Let G be a graph. Two subsets D_1 and D_2 in the graph G are distinguishable under the MM* model iff one of the following conditions holds (see Fig. 3):*

(1) *There exist $yz \in E(G)$ and $xz \in E(G)$ with $x, z \in V(G - D_1 - D_2)$ and $y \in (D_1 - D_2) \cup (D_2 - D_1)$;*

(2) *There exist $yz \in E(G)$ and $xz \in E(G)$ with $x, y \in D_1 - D_2$ and $z \in V(G - (D_1 \cup D_2))$;*

(3) *There exist $yz \in E(G)$ and $xz \in E(G)$ with $x, y \in D_2 - D_1$ and $z \in V(G - (D_1 \cup D_2))$.*

Let the g-good-neighbor diagnosability of S_n^2 under the MM* model be $t_g^m(S_n^2)$. We then show that the upper bound of $t_g^m(S_n^2)$ by construction.

Theorem 2. *Let S_n^2 ($n \geq 5$) be an n-dimensional split-star network. The upper bound of $t_g^m(S_n^2)$ is $2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$.*

Proof. For $1 \leq g \leq 3$, let $\mathbb{M}_g$ be a subgraph of S_n^2 such that $\mathbb{M}_1$ is a 1-path P_1 in $S_{n,E}^2$, $\mathbb{M}_2$ is a 3-cycle C_3 in $S_{n,E}^2$, and $\mathbb{M}_3$ is a 3-dimensional split-star subnetwork S_3^2 in S_n^2 for $n \geq 5$. Let $D_1 = N_{S_n^2}(V(\mathbb{M}_g))$ and $D_2 = N_{S_n^2}(V(\mathbb{M}_g)) \cup V(\mathbb{M}_g)$ be two faulty vertex-sets of S_n^2 (see Fig. 4). By Lemma 2, $|N_{S_n^2}(V(\mathbb{M}_1))| = 4n - 9$ and $|N_{S_n^2}(V(\mathbb{M}_2))| = 6n - 15$. By Theorem 1, $|N_{S_n^2}(V(\mathbb{M}_3))| = 12n - 36$. Hence, we use the uniform expression to represent $|N_{S_n^2}(V(\mathbb{M}_g))|$ for $1 \leq g \leq 3$ as follows:

$$|D_1| = |N_{S_n^2}(V(\mathbb{M}_g))| = 2 \times (2^g - g + 1) \times n - 15/2 \times g^2 + 33/2 \times g - 18.$$

Since $N_{S_n^2}(V(\mathbb{M}_g)) \cap V(\mathbb{M}_g) = \emptyset$, hence

$$\begin{aligned} |D_2| &= |N_{S_n^2}(V(\mathbb{M}_g)) \cup V(\mathbb{M}_g)| \\ &= |N_{S_n^2}(V(\mathbb{M}_g))| + |V(\mathbb{M}_g)| - |N_{S_n^2}(V(\mathbb{M}_g)) \cap V(\mathbb{M}_g)| \\ &= |N_{S_n^2}(V(\mathbb{M}_g))| + |V(\mathbb{M}_g)| \\ &= 2 \times (2^g - g + 1) \times n - 15/2 \times g^2 + 33/2 \times g - 18 + (g^2 - 2 \times g + 3) \\ &= 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 15, \end{aligned}$$

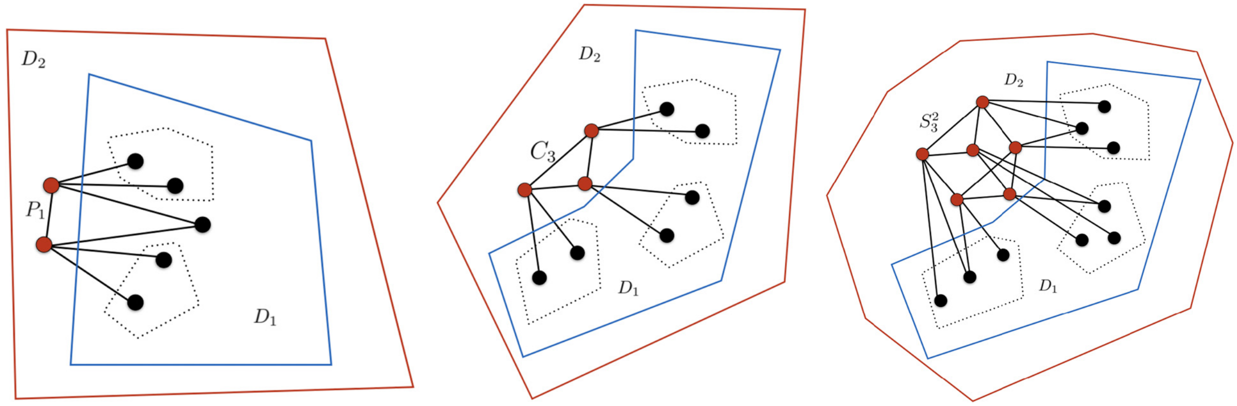

Fig. 4. The illustrations of $D_1 = N_{S_n^2}(V(\mathbb{M}_g))$ and $D_2 = N_{S_n^2}[V(\mathbb{M}_g)]$.

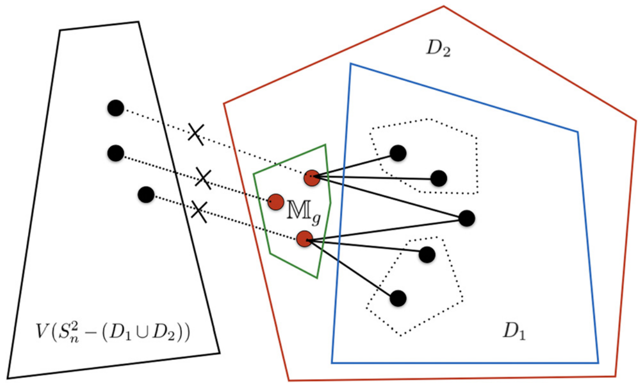

Fig. 5. An illustration of $E[V(S_n^2 - (D_1 \cup D_2)), (D_1 - D_2) \cup (D_2 - D_1)] = \emptyset$.

and the minimum degree of $S_n^2 - D_2$ is $\delta(S_n^2 - D_2) \geq g$ for $1 \leq g \leq 3$.

By Definition 3 (1), D_1 and D_2 are two g-good-neighbor faulty vertex-sets of S_n^2 with

$$|D_1| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 15, |D_2| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 15.$$

Moreover, since $V(\mathbb{M}_g) = D_2 - D_1$, $N_{S_n^2}(V(\mathbb{M}_g)) = D_1$ and $D_1 \subset D_2$, there is no edge between $V(S_n^2 - (D_1 \cup D_2))$ and $(D_1 - D_2) \cup (D_2 - D_1)$ (see Fig. 5). By Lemma 6, D_1 and D_2 are two indistinguishable faulty vertex-sets of S_n^2 under the MM* model. By Definition 3 (2), the split-star network S_n^2 is not g-good-neighbor $(2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 15)$-diagnosable under the MM* model. By Definition 3 (3), the upper bound of the g-good-neighbor diagnosability of S_n^2 under the MM* model is

$$t_g^m(S_n^2) \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$$

for $n \geq 5$ and $1 \leq g \leq 3$. Hence, the theorem holds. □

Lemma 7. [23] *Let D be a faulty vertex-set of S_n^2 with $|D| \leq 6n - 17$. Then, $S_n^2 - D$ $(n \geq 4)$ has a large component C and $|V(S_n^2 - D - C)| \leq 2$.*

Lemma 8. [23] *Let D be a faulty vertex-set of S_n^2 with $|D| \leq 8n - 25$. Then, $S_n^2 - D$ $(n \geq 4)$ has a large component C and $|V(S_n^2 - D - C)| \leq 3$.*

Theorem 3. *Let S_n^2 $(n \geq 5)$ be an n-dimensional split-star network. Let D_1 and D_2 be two indistinguishable g-good-neighbor faulty vertex-sets of S_n^2 under the MM* model with $|D_1| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$, $|D_2| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$, and $1 \leq g \leq 3$. If $V(S_n^2) \neq D_1 \cup D_2$, then $S_n^2 - (D_1 \cup D_2)$ has no isolated vertex.*

Proof. If $2 \leq g \leq 3$, then D_1 is a g-good-neighbor faulty vertex-set of S_n^2. Hence, $|N_{S_n^2 - D_1}(x)| \geq g \geq 2$ for any $x \in V(S_n^2 - D_1)$. The faulty vertex-sets D_1 and D_2 of S_n^2 do not satisfy any condition in Lemma 6. Hence, $|N_{S_n^2[D_2 - D_1]}(w)| \leq 1$ for any

$w \in V(S_n^2-(D_1 \cup D_2))$. Thus, for any $w \in V(S_n^2-(D_1 \cup D_2))$, the number of all neighbors of w in the subgraph $S_n^2-(D_1 \cup D_2)$ is equal to the number of all neighbors of w in the subgraph $S_n^2 - D_1$ minus the number of all neighbors of w in the subgraph $S_n^2[D_2 - D_1]$. Hence,

$$|N_{S_n^2-(D_1\cup D_2)}(w)| = |N_{S_n^2-D_1}(w)| - |N_{S_n^2[D_2-D_1]}(w)| \geq g-1 \geq 1 \text{ for } 2 \leq g \leq 3.$$

By the arbitrariness of $w \in V(S_n^2-(D_1 \cup D_2))$, every vertex of $V(S_n^2-(D_1 \cup D_2))$ is not isolated. Hence, the theorem holds.

If $g = 1$ and $D_1 \subset D_2$, then $S_n^2-(D_1 \cup D_2) = S_n^2 - D_2$. Because D_2 is a 1-good-neighbor faulty vertex-set of S_n^2, $S_n^2 - D_2$ has no isolated vertices. When $D_1 \not\subseteq D_2$, suppose that there exists one isolated vertex in $S_n^2-(D_1 \cup D_2)$. Let $W \subseteq V(S_n^2-(D_1 \cup D_2))$ consist of isolated vertices and let $H = V(S_n^2-(D_1 \cup D_2)-W)$. Let $w \in W$. Since D_1 is a 1-good-neighbor faulty vertex-set of S_n^2, thus there exists a vertex u in $D_2 - D_1$ with $uw \in E(S_n^2)$. Moreover, the faulty vertex-sets D_1 and D_2 of S_n^2 do not satisfy Lemma 6, hence there exists just one vertex $u \in D_2 - D_1$ with $uw \in E(S_n^2)$. In the same way, there exists just one vertex $v \in D_1 - D_2$ with $vw \in E(S_n^2)$. Hence, $|N_{S_n^2[D_1\cap D_2]}(w)| = 2n-5$ for $w \in W$ by Lemma 1 (4). Since $|D_2| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ and $g = 1$, we have $|D_2| \leq 4n - 8$.

Therefore, the number of all neighbors of any vertex w in the vertex-set W in the subgraph $S_n^2[D_1 \cap D_2]$ is equal to the size of W multiply by $2n - 5$. Hence,

$$\begin{aligned}
&\textstyle\sum_{w\in W} |N_{S_n^2[D_1\cap D_2]}(w)| \\
&= \textstyle\sum_{w\in W} [|N_{S_n^2}(w)| - |N_{S_n^2[D_1-D_2)}(w)| - |N_{S_n^2[D_2-D_1]}(w)|] \\
&= |W|(|N_{S_n^2}(w)| - 2) \\
&= |W|(2n-5).
\end{aligned}$$

Moreover, the number of all neighbors of any vertex w in the vertex-set W in the subgraph $S_n^2[D_1 \cap D_2]$ is no more than the number of all neighbors of any vertex v in the vertex-set $D_1 \cap D_2$ in the subgraph S_n^2. Thus,

$$\begin{aligned}
\textstyle\sum_{w\in W} |N_{S_n^2[D_1\cap D_2]}(w)| &\leq \textstyle\sum_{v\in D_1\cap D_2} |N_{S_n^2}(v)| \\
&\leq |D_1 \cap D_2|(2n-3) \\
&\leq (|D_2| - |D_1|)(2n-3) \\
&\leq (|D_2| - 1)(2n-3) \\
&\leq (4n-9)(2n-3).
\end{aligned}$$

Therefore, $|W|(2n-5) \leq (4n-9)(2n-3)$. It follows that $|W| \leq 4n-4$ for $n \geq 5$.

Assume that $H = \emptyset$, then $V(S_n^2) = (D_1 \cup D_2) \cup W$. Since $(D_1 \cup D_2) \cap W = \emptyset$, for $n \geq 5$ and $g = 1$, the proof that the size of S_n^2 is less than $n!$ is listed as follows:

$$\begin{aligned}
n! &= |V(S_n^2)| \\
&= |(D_1 \cup D_2) \cup W| \\
&= |(D_1 \cup D_2)| + |W| - |(D_1 \cup D_2) \cap W| \\
&\leq |D_1| + |D_2| - |D_1 \cap D_2| + |W| \\
&\leq 2 \times (4n-8) - (2n-5) + (4n-4) \\
&< n!,
\end{aligned}$$

which contradicts to $|V(S_n^2)| = n!$. Hence, $H \neq \emptyset$. Since the faulty vertex-sets D_1 and D_2 of S_n^2 do not satisfy the condition (1) of Lemma 6 and $V(H)$ has no isolated vertex, there is no edge between H and $(D_1 - D_2) \cup (D_2 - D_1)]$. Therefore, $D_1 \cap D_2$ is a 1-restricted vertex-cut of S_n^2 (see Fig. 6).

By Lemma 2 (1), $|D_1 \cap D_2| \geq 4n - 9$. Note that $|D_1| \leq 4n-8$, $|D_2| \leq 4n-8$ and $D_1 - D_2 \neq \emptyset$, $D_2 - D_1 \neq \emptyset$. Thus, $|D_1 - D_2| = |D_2 - D_1| = 1$. Let $D_1 - D_2 = \{v_1\}$ and $D_2 - D_1 = \{v_2\}$. Hence, $wv_1 \in E(S_n^2)$ and $wv_2 \in E(S_n^2)$ for any $w \in W$. By Lemma 1 (4), $|N_{S_n^2}(v_1) \cap N_{S_n^2}(v_2)| \leq 2$. Hence, $1 \leq |W| \leq 2$.

When $|W| = 1$, by Lemma 7, we have $|D_1 \cap D_2| \geq 6n - 16$ (see Fig. 7). Hence, for $n \geq 5$, $|D_2| = |D_2 - D_1| + |D_1 \cap D_2| \geq 6n - 15 > 4n - 8 \geq |D_2|$, which is a contradiction.

If $|W| = 2$, by Lemma 8, $|D_1 \cap D_2| \geq 8n - 24$ (see Fig. 8). Hence, for $n \geq 5$, $|D_2| = |D_2 - D_1| + |D_1 \cap D_2| \geq 8n - 23 > 4n - 8 \geq |D_2|$, which is a contradiction.

Therefore, the theorem holds. □

Next, we will prove that the lower bound of $t_g^m(S_n^2)$ is $2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$ by the above theorems.

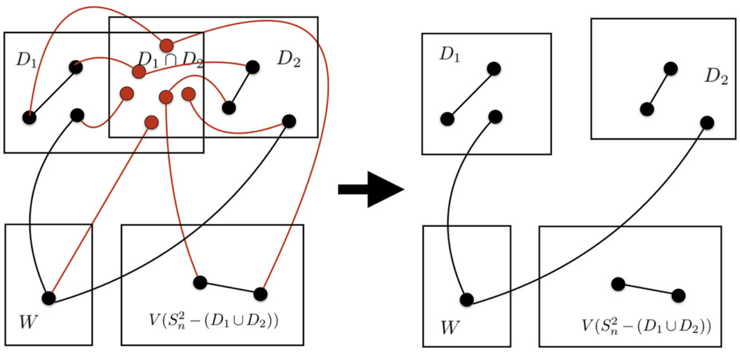

Fig. 6. An illustration that $D_1 \cap D_2$ is a 1-restricted vertex-cut of S_n^2 when $W \neq \emptyset$.

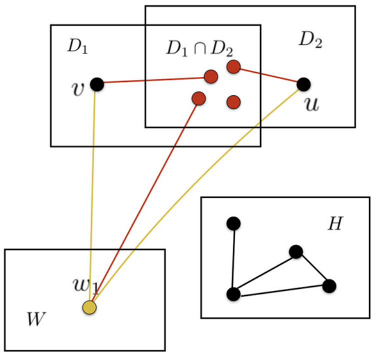

Fig. 7. An illustration of $|D_1 \cap D_2| \geq 6n - 16$.

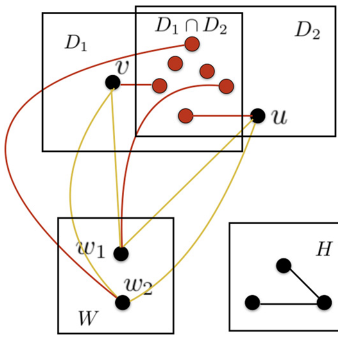

Fig. 8. An illustration of $|D_1 \cap D_2| \geq 8n - 24$.

Theorem 4. *Let S_n^2 ($n \geq 5$) be an n-dimensional split-star network. The lower bound of $t_g^m(S_n^2)$ is $2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$.*

Proof. Suppose that $t_g^m(S_n^2) < 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $n \geq 5$ and $1 \leq g \leq 3$ under the MM* model. By Definitions 3 (2)–(3), there exist two indistinguishable g-good-neighbor faulty vertex-sets D_1 and D_2 of S_n^2 with

$$|D_1| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16,$$

$$|D_2| \leq 2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16.$$

By Lemma 6, the faulty vertex-sets D_1 and D_2 of S_n^2 do not satisfy Lemma 6. Assume that $D_2 - D_1 \neq \emptyset$. If $V(S_n^2) = D_1 \cup D_2$, then

$$n! = |V(S_n^2)| \leq |D_1| + |D_2| \leq 2[2 \times (2^g - g + 1) \times n - 13/2 \times g^2 + 29/2 \times g - 16] < n!$$

for $n \geq 5$ and $1 \leq g \leq 3$, which is a contradiction.

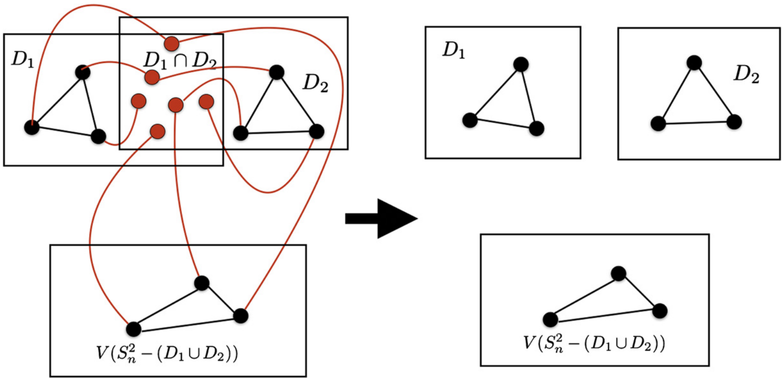

Fig. 9. An illustration that $D_1 \cap D_2$ is also a g-restricted vertex-cut of S_n^2.

Hence, $V(S_n^2) \neq D_1 \cup D_2$. Let u be a vertex in $S_n^2-(D_1 \cup D_2)$. By Theorem 3, there exists at least one vertex v in $S_n^2-(D_1 \cup D_2)$ such that $uv \in E(S_n^2)$. Since the faulty vertex-sets D_1 and D_2 of S_n^2 do not satisfy Lemma 6, u and v have no neighbor in $(D_1-D_2)\cup(D_2-D_1)$. By the arbitrariness of $u \in V(S_n^2-(D_1\cup D_2))$, there is no edge between $V(S_n^2-(D_1\cup D_2))$ and $(D_1-D_2)\cup(D_2-D_1)$.

Since D_1 is a g-good-neighbor faulty vertex-set of S_n^2 and $D_2-D_1 \neq \emptyset$, $\delta(S_n^2[D_2-D_1]) \geq g^2-2g+3$. Hence, $|D_2-D_1| \geq g+1$. Since D_1 and D_2 are two g-good-neighbor faulty vertex-sets of S_n^2 and there is no edge between $V(S_n^2-(D_1\cup D_2))$ and $(D_1-D_2)\cup(D_2-D_1)$, $D_1 \cap D_2$ is also a g-restricted vertex-cut of S_n^2 for $1 \leq g \leq 3$ (see Fig. 9). By Lemma 2 and Theorem 1,

$$|D_1 \cap D_2| \geq 2 \times (2^g-g+1) \times n - 15/2 \times g^2 + 33/2 \times g - 18.$$

Therefore,

$$\begin{aligned} |D_2| &= |D_2-D_1| + |D_1 \cap D_2| \\ &\geq (g^2-2g+3) + (2 \times (2^g-g+1) \times n - 15/2 \times g^2 + 33/2 \times g - 18) \\ &= 2 \times (2^g-g+1) \times n - 13/2 \times g^2 + 29/2 \times g - 15, \end{aligned}$$

which contradicts to $|D_2| \leq 2 \times (2^g-g+1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$.

In conclusion, the lower bound of $t_g^m(S_n^2)$ is $2 \times (2^g-g+1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$. Hence, the theorem holds. □

According to Theorems 2 and 4, the following theorem holds.

Theorem 5. *Let S_n^2 $(n \geq 5)$ be an n-dimensional split-star network. Then $t_g^m(S_n^2) = 2 \times (2^g-g+1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$.*

5. Conclusion

In this paper, we propose a simple and complete proof to show that, under the MM* model, the g-good-neighbor diagnosability of S_n^2 is $2 \times (2^g-g+1) \times n - 13/2 \times g^2 + 29/2 \times g - 16$ for $1 \leq g \leq 3$, which is several times higher than the original diagnosability. In the future, we will establish a universal method for the general g-restricted connectivity and the g-good-neighbor diagnosability of S_n^2 where $g \geq 4$.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work was supported by the Fok Ying Tung Education Foundation (No. 171061), National Natural Science Foundation of China (No. 61702100, No. U1905211, No. 61702103, No. 61771140).

References

[1] S.B. Akers, D. Harel, B. Krishnamurthy, The star graph: an attractive alternative to the n-cube, in: Proceedings of the International Conference on Parallel Processing, 1987, pp. 393–400.
[2] N.-W. Chang, S.-Y. Hsieh, Conditional diagnosability of augmented cubes under the PMC model, IEEE Trans. Dependable Secure Comput. 9 (1) (2012) 46–60.
[3] C.-A. Chen, S.-Y. Hsieh, (t, k)-Diagnosis for component-composition graphs under the MM* model, IEEE Trans. Comput. 60 (12) (2011) 1704–1717.
[4] D. Cheng, A relationship between g-good-neighbour conditional diagnosability and g-good-neighbour connectivity in regular graphs, Int. J. Comput. Math.: Comput. Syst. Theory 3 (2018) 47–52.
[5] E. Cheng, M.J. Lipman, Orienting split-stars and alternating group graphs, Networks 35 (2) (2000) 139–144.
[6] E. Cheng, M.J. Lipman, Vulnerability issues of star graphs, alternating group graphs and split-stars: strength and toughness, Discrete Appl. Math. 118 (2002) 163–179.
[7] E. Cheng, M.J. Lipman, H.A. Park, An attractive variation of the star graphs: split-stars, Technical Report 98 (3), 1998.
[8] E. Cheng, M.J. Lipman, H.A. Park, Super connectivity of star graphs, alternating group graph networks and split-stars, Ars Combinatoria 59 (2001) 107–116.
[9] E. Cheng, L. Lipták, F. Sala, Linearly many faults in 2-tree-generated networks, Networks 55 (2010) 90–98.
[10] E. Cheng, K. Qiu, Z. Shen, A general approach to deriving the g-good-neighbor conditional diagnosability of interconnection networks, Theor. Comput. Sci. 757 (2019) 56–67.
[11] J. Fan, Diagnosability of the Möbius cubes, IEEE Trans. Parallel Distrib. Syst. 9 (9) (1998) 923–928.
[12] J. Fan, Diagnosability of crossed cubes under the comparison diagnosis model, IEEE Trans. Parallel Distrib. Syst. 13 (7) (2002) 687–692.
[13] M.-M. Gu, R.-X. Hao, D.-X. Yang, A short note on the 1, 2-good-neighbor diagnosability of balanced hypercubes, J. Interconnect. Netw. 16 (02) (2016) 1650001.
[14] M.-M. Gu, R.-X. Hao, A.-M. Yu, The 1-good-neighbor conditional diagnosability of some regular graphs, J. Interconnect. Netw. 17 (3n04) (2017) 1741001.
[15] J. Guo, D. Li, M. Lu, The g-good-neighbor conditional diagnosability of the crossed cubes under the PMC and MM* model, Theor. Comput. Sci. 755 (2019) 81–88.
[16] R.-X. Hao, Y.-Q. Feng, J.-X. Zhou, Conditional diagnosability of alternating group graphs, IEEE Trans. Comput. 62 (4) (2013) 827–831.
[17] X. Hu, W. Yang, Y. Tian, J. Meng, Equal relation between g-good-neighbor diagnosability under the PMC model and g-good-neighbor diagnosability under the MM* model of a graph, Discrete Appl. Math. 262 (2019) 96–103.
[18] Y. Huang, L. Lin, L. Xu, X. Wang, Extra diagnosability and good-neighbor diagnosability of n-dimensional alternating group graph AG_n under the PMC model, Theor. Comput. Sci. 795 (2019) 36–49, https://doi.org/10.1016/j.tcs.2019.05.034.
[19] J.S. Jwo, S. Lakshmivarahan, S.K. Dhall, A new class of interconnection networks based on the alternating group, Networks 23 (1993) 315–326.
[20] S. Latifi, M. Hegde, M. Naraghi-Pour, Conditional connectivity measures for large multiprocessor systems, IEEE Trans. Comput. 43 (2) (1994) 218–222.
[21] D. Li, M. Lu, The g-good-neighbor conditional diagnosability of star graphs under the PMC and MM* model, Theor. Comput. Sci. 674 (2017) 53–59.
[22] L. Lin, S.-Y. Hsieh, R. Chen, L. Xu, C.-W. Lee, The relationship between g-restricted connectivity and g-good-neighbor fault diagnosability of general regular networks, IEEE Trans. Reliab. 67 (1) (2018) 285–296.
[23] L. Lin, L. Xu, S. Zhou, Conditional diagnosability and strong diagnosability of Split-Star Networks under the PMC model, Theor. Comput. Sci. 562 (2015) 565–580.
[24] L. Lin, L. Xu, S. Zhou, S.-Y. Hsieh, The extra, restricted connectivity and conditional diagnosability of split-star networks, IEEE Trans. Parallel Distrib. Syst. 27 (2) (2016) 533–545.
[25] L. Lin, L. Xu, D. Wang, S. Zhou, The g-good-neighbor conditional diagnosability of arrangement graphs, IEEE Trans. Dependable Secure Comput. 15 (3) (2018) 542–548.
[26] J. Maeng, M. Malek, A comparison connection assignment for self-diagnosis of multiprocessors systems, in: Proceedings of the 11th International Symposium on Fault-Tolerant Computing, 1981, pp. 173–175.
[27] S.-L. Peng, C.-K. Lin, J.J.M. Tan, L.-H. Hsu, The g-good-neighbor conditional diagnosability of hypercube under PMC model, Appl. Math. Comput. 218 (2012) 10406–10412.
[28] A. Sengupta, A. Dahbura, On self-diagnosable multiprocessor systems: diagnosis by the comparison approach, IEEE Trans. Comput. 41 (11) (1992) 1386–1396.
[29] S. Wang, W. Han, The g-good-neighbor conditional diagnosability of n-dimensional hypercubes under the MM* model, Inf. Process. Lett. 116 (9) (2016) 574–577.
[30] S. Wang, M. Wang, The g-good-neighbor and g-extra diagnosability of networks, Theor. Comput. Sci. 773 (2019) 107–114.
[31] Y. Wei, M. Xu, On g-good-neighbor conditional diagnosability of (n, k)-star networks, Theor. Comput. Sci. 697 (2017) 79–90.
[32] Y. Wei, M. Xu, The 1, 2-good-neighbor conditional diagnosabilities of regular graphs, Appl. Math. Comput. 334 (2018) 295–310.
[33] X. Xu, X. Li, S. Zhou, R.-X. Hao, M.-M. Gu, The g-good-neighbor diagnosability of (n, k)-star graphs, Theor. Comput. Sci. 659 (2017) 53–63.
[34] J. Yuan, A. Liu, X. Ma, X. Liu, X. Qin, J. Zhang, The g-good-neighbor conditional diagnosability of k-ary n-cubes under the PMC model, and comparison model, IEEE Trans. Parallel Distrib. Syst. 26 (4) (2015) 1165–1177.
[35] Y. Zhai, L. Lin, L. Xu, X. Zhang, Y. Huang, The conditional diagnosability with g-good-neighbor of exchanged hypercubes, Comput. J. 62 (5) (2019) 747–756.